Hjem 4 løsningsskitse

a) Maksimalt antal lineært uafh vektorer i V er 8, dimensionen af ethvert underrum ≤ 8.

b) $$\begin{bmatrix} | & | & | \\ \underline{v}_1 & \underline{v}_2 & \underline{v}_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 2 & 3 & 0 \\ -2 & 4 & -7 \\ 0 & 5 & -5 \\ 4 & 0 & 6 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

En mulig basis kunne være $(\underline{v}_1, \underline{v}_2)$ så $\dim_{\mathbb{R}}(W) = 2$

c) Lad α betegne standard basis for $\mathbb{C}^2$.

$$_\alpha[L]_\gamma = \begin{bmatrix} | & | & | \\ _\alpha L\begin{pmatrix}1\\1\\1\end{pmatrix} & _\alpha L\begin{pmatrix}0\\1\\1\end{pmatrix} & _\alpha L\begin{pmatrix}0\\0\\1\end{pmatrix} \\ | & | & | \end{bmatrix} = \begin{bmatrix} 6 & 5 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

$$_\alpha[id]_\beta = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad _\beta[id]_\alpha = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$_\beta[L]_\gamma = {_\beta[id]_\alpha}\,{_\alpha[L]_\gamma} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 & 5 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 2 \\ -2 & -1 & -1 \end{bmatrix}$$

d) $$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}_3) = -\lambda(-1-\lambda)(2-\lambda) + 1\cdot(-1-\lambda)$$
$$= (\lambda+1)(-\lambda^2+2\lambda-1) = -(\lambda+1)(\lambda-1)^2$$

Egenværdier $\lambda_1 = 1$, $\lambda_2 = -1$

$$E_{\lambda_2} = \operatorname{span}_{\mathbb{R}}\begin{pmatrix}0\\1\\0\end{pmatrix}, \quad E_{\lambda_1} = \operatorname{span}_{\mathbb{R}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

Baser er klart fra ovenstående, kan ikke diagonaliseres e)

1.) Første søjle i afb. matricen findes som:

$${}_\beta M\left(\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\right) = {}_\beta\left(\begin{bmatrix}4 & 0\\ 2 & 0\end{bmatrix}\right) = \begin{pmatrix}4\\2\\0\\0\end{pmatrix}$$

Tilsvarende for de øvrige:

$${}_\beta[M]_\beta = \begin{bmatrix}4 & 6 & 0 & 0\\ 2 & 3 & 0 & 0\\ 0 & 0 & 4 & 6\\ 0 & 0 & 2 & 3\end{bmatrix}$$

2.) $$\begin{bmatrix}4 & 6 & 0 & 0\\ 2 & 3 & 0 & 0\\ 0 & 0 & 4 & 6\\ 0 & 0 & 2 & 3\end{bmatrix} \longrightarrow \begin{bmatrix}1 & 3/2 & 0 & 0\\ 0 & 0 & 1 & 3/2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Pivot element i søjle 1 og 3, så billedet er udspændt af vektorerne:

$${}_\beta\underline{v}_1 = \begin{pmatrix}4\\2\\0\\0\end{pmatrix} \text{ og } {}_\beta\underline{v}_2 = \begin{pmatrix}0\\0\\4\\2\end{pmatrix}$$

$$\operatorname{Im}(M) = \operatorname{span}_{\mathbb{R}}\left(\begin{bmatrix}4 & 0\\ 2 & 0\end{bmatrix}, \begin{bmatrix}0 & 4\\ 0 & 2\end{bmatrix}\right)$$

$$\dim_{\mathbb{R}}(\operatorname{Im}(M)) = 2$$

$$\ker({}_\beta[M]_\beta) = \operatorname{span}_{\mathbb{R}}\left(\begin{pmatrix}-3/2\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-3/2\\1\end{pmatrix}\right)$$

$$\ker(M) = \operatorname{span}_{\mathbb{R}}\left(\begin{bmatrix}-3/2 & 0\\ 1 & 0\end{bmatrix}, \begin{bmatrix}0 & -3/2\\ 0 & 1\end{bmatrix}\right)$$

$$\dim(\ker(M)) = 2.$$

Istedet for at bestemme kernen kan dimensionssætningen anvendes.